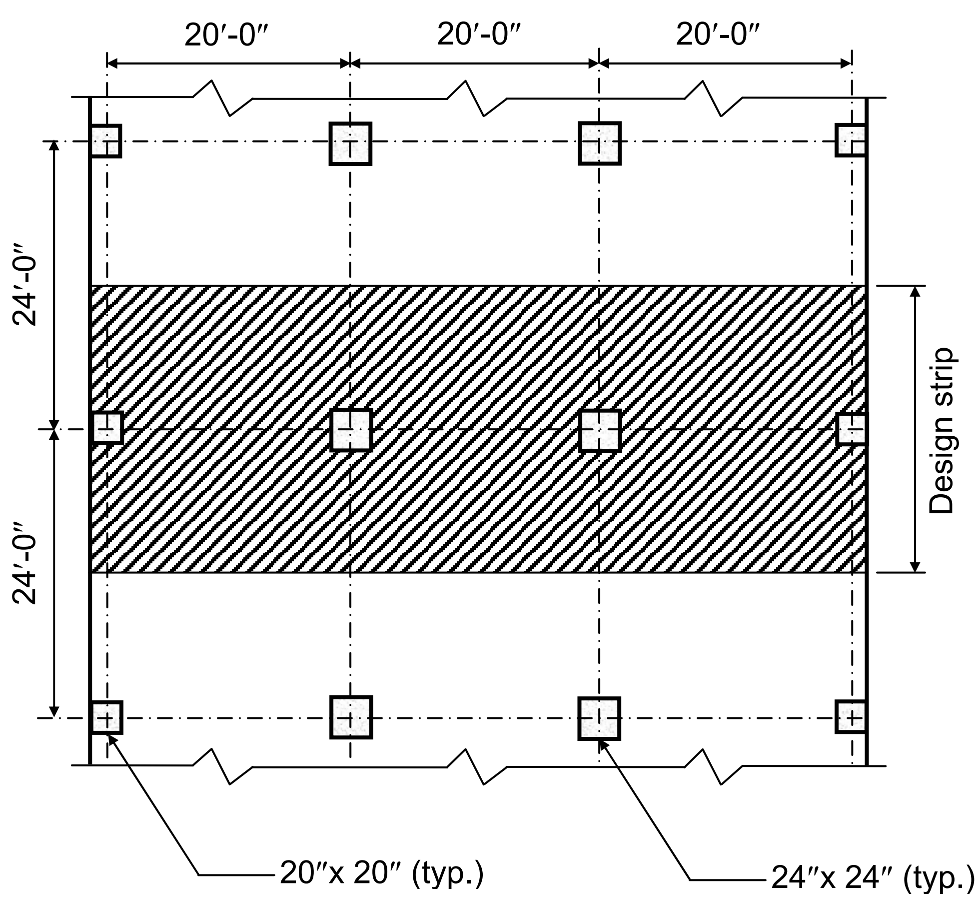
The following example illustrates the design methods presented in the article "Timesaving Design Aids for Reinforced Concrete, Part 2: Two-way Slabs," by David A. Fanella, which appeared in the October 2001 edition of Structural Engineer *magazine. Unless otherwise noted, all referenced table, figure, and equation numbers are from that article.*

Example Building

Below is a partial plan of a typical floor in a cast-in-place reinforced concrete building. In this example, an interior strip of a flat plate floor system is designed and detailed for the effects of gravity loads according to ACI 318-99.



Design Data

Materials

- Concrete: normal weight (150 pcf), ¾-in. maximum aggregate, f'_c = 4,000 psi
- Mild reinforcing steel: Grade 60 (f_y = 60,000 psi)

Loads

- Superimposed dead loads = 30 psf
- Live load = 50 psf

Minimum Slab Thickness

Longest clear span ℓ_n = 24 − (20/12) = 22.33 ft

From Fig. 1, minimum thickness h per ACI Table 9.5(c) = ℓ_n/30 = 8.9 in.

Use Fig. 2 to determine h based on shear requirements at edge column assuming a 9 in. slab:

w_u = 1.4(112.5 + 30) + 1.7(50) = 284.5 psf

A = 24 x [(20 + 1.67)/2] = 260 ft^2

A/c_1^2 = 260/1.67^2 = 93.6

From Fig. 2, $d/c_1 \approx 0.39$

d = 0.39 x 20 = 7.80 in.

h = 7.80 + 1.25 = 9.05 in.

Try preliminary h = 9.0 in.

Design for Flexure

Use Fig. 3 to determine if the Direct Design Method of ACI Sect. 13.6 can be utilized to compute the bending moments due to the gravity loads:

- 3 continuous spans in one direction, more than 3 in the other   O.K.
- Rectangular panels with long-to-short span ratio = 24/20 = 1.2 < 2   O.K.
- Successive span lengths in each direction are equal   O.K.
- No offset columns   O.K.
- L/D = 50/(112.5 + 30) = 0.35 < 2   O.K.
- Slab system has no beams   N.A.

Since all requirements are satisfied, the Direct Design Method can be used.

Total panel moment M_o in end span:

$$M_o = \frac{w_u \ell_2 \ell_n^2}{8} = \frac{0.285 \times 24 \times 18.167^2}{8}$$

$$= 282.2 \text{ ft - kips}$$

Total panel moment M_o in interior span:

$$M_o = \frac{w_u \ell_2 \ell_n^2}{8} = \frac{0.285 \times 24 \times 18.0^2}{8}$$

$$= 277.0 \text{ ft - kips}$$

For simplicity, use M_o = 282.2 ft-kips for all spans.

Division of the total panel moment M_o into negative and positive moments, and then column and middle strip moments, involves the direct application of the moment coefficients in Table 1.

Slab Moments (ft-kips)	End Spans			Int. Span
	Ext. neg.	Positive	Int. neg.	Positive
Total Moment	73.4	146.7	197.5	98.8
Column Strip	73.4	87.5	149.6	59.3
Middle Strip	0	59.3	48.0	39.5

Note: All negative moments are at face of support.

Required slab reinforcement.

Span Location		M_u (ft-kips)	b* (in.)	d** (in.)	A_s† (in.^{2})	Min. A_s‡ (in.^{2})	Reinforcement$^+$
End Span							
Column Strip	Ext. neg.	73.4	120	7.75	2.37	1.94	12-No. 4
	Positive	87.5	120	7.75	2.82	1.94	15-No. 4
	Int. Neg.	149.6	120	7.75	4.83	1.94	25-No. 4
Middle Strip	Ext. neg.	0.0	168	7.75	---	2.72	14-No. 4
	Positive	59.3	168	7.75	1.91	2.72	14-No. 4
	Int. Neg.	48.0	168	7.75	1.55	2.72	14-No. 4
Interior Span							
Column Strip	Positive	59.3	120	7.75	1.91	1.94	10-No. 4
Middle Strip	Positive	39.5	168	7.75	1.27	2.72	14-No. 4

*Column strip width b = (20 x 12)/2 = 120 in.
Middle strip width b = (24 x 12) – 120 = 168 in.
**Use average d = 9 – 1.25 = 7.75 in.
†$A_s = M_u/4d$ where M_u is in ft-kips and d is in inches
‡Min. A_s = 0.0018bh = 0.0162b; Max. s = 2h = 18 in. or 18 in. (Sect. 13.3.2)
$^+$For maximum spacing: 120/18 = 6.7 spaces, say 8 bars
168/18 = 9.3 spaces, say 11 bars

Design for Shear

Check slab shear and flexural strength at edge column due to direct shear and unbalanced moment transfer.

Check slab reinforcement at exterior column for moment transfer between slab and column.

Portion of total unbalanced moment transferred by flexure = $\gamma_f M_u$

$b_1 = 20 + (7.75/2) = 23.875$ in.

$b_2 = 20 + 7.75 = 27.75$ in.

$b_1 / b_2 = 0.86$

From Fig. 5, $\gamma_f = 0.62$*

$\gamma_f M_u = 0.62 \times 73.4 = 45.5$ ft-kips

Required $A_s = 45.5/(4 \times 7.75) = 1.47$ in.^{2}

Number of No. 4 bars = 1.47/0.2 = 7.4, say 8 bars

Must provide 8-No. 4 bars within an effective slab width = $3h + c_2 = (3 \times 9) + 20 = 47$ in.

Provide the required 8-No. 4 bars by concentrating 8 of the column strip bars (12-No. 4) within the 47 in. slab width over the column.

Check bar spacing:

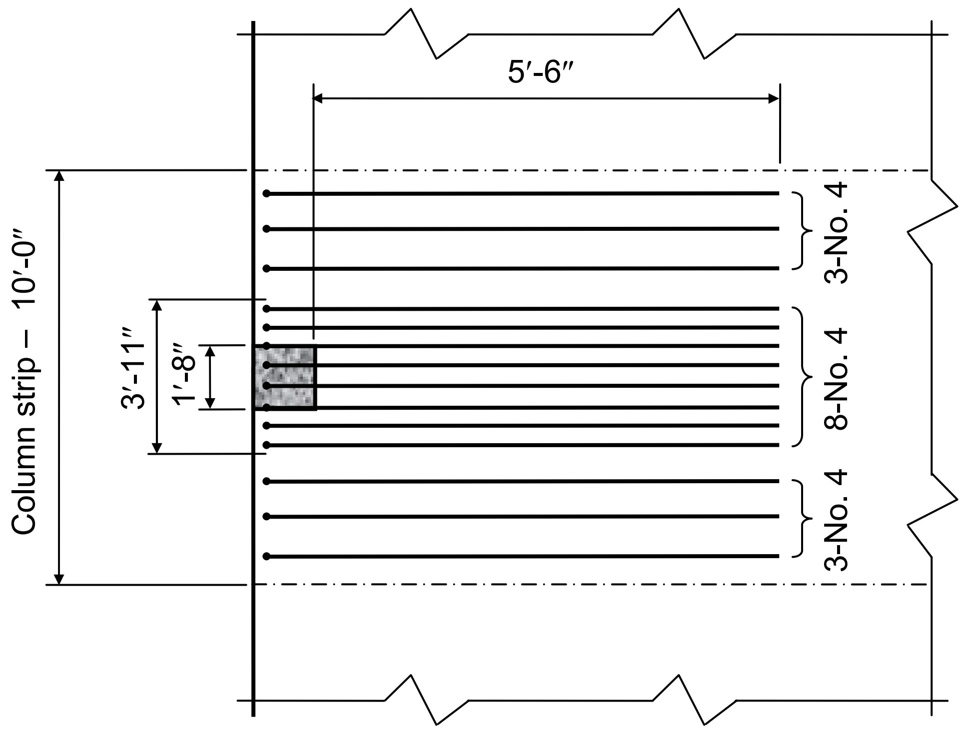
For 8-No. 4 within 47 in. width: 47/8 = 5.9 in. < 18 in. O.K.

For 4-No. 4 within 120 – 47 = 73 in. width: 73/4 = 18.25 in. > 18 in.

Add 1 additional bar on each side of the 47 in. strip; the spacing becomes 73/6 = 12.2 in. < 18 in. O.K.

Reinforcement details at this location are shown in the figure on the next page (see Fig. 6).

*The provisions of Sect. 13.5.3.3 may be utilized; however, they are not in this example.



Check the combined shear stress at the inside face of the critical transfer section.

$$v_u = \frac{V_u}{A_c} + \frac{\gamma_v M_u}{J/c}$$

Factored shear force at edge column:

$V_u = 0.285[(24 \times 10.83) - (1.99 \times 2.31)]$
$= 72.8$ kips

When the end span moments are determined from the Direct Design Method, the fraction of unbalanced moment transferred by eccentricity of shear must be $0.3M_o = 0.3 \times 282.2 = 84.7$ ft-kips (Sect. 13.6.3.6).

$\gamma_v = 1 - \gamma_f = 1 - 0.62 = 0.38$

$c_2/c_1 = 1.0$

$c_1/d = 20/7.75 = 2.58$

Interpolating from Table 7, $f_1 = 9.74$ and $f_2 = 5.53$

$A_c = f_1 d^2 = 9.74 \times 7.75^2 = 585.0$ in.^{2}

$J/c = 2f_2d^3 = 2 \times 5.53 \times 7.75^3 = 5{,}148 \text{ in.}^3$

$$v_u = \frac{72{,}800}{585.0} + \frac{0.38 \times 84.7 \times 12{,}000}{5{,}148}$$

$$v_u = 124.4 + 75.0 = 199.4 \text{ psi}$$

Determine allowable shear stress ϕv_c from Fig. 4b:

$b_o/d = (2b_1 + b_2)/d$

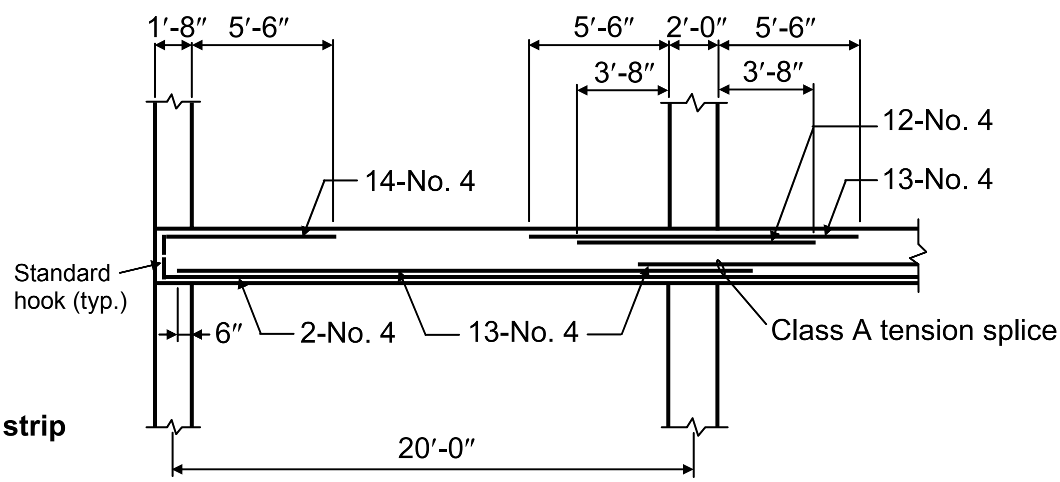
$b_o/d = [(2 \times 23.875) + 27.75]/7.75 = 9.74$

$\beta_c = 1$

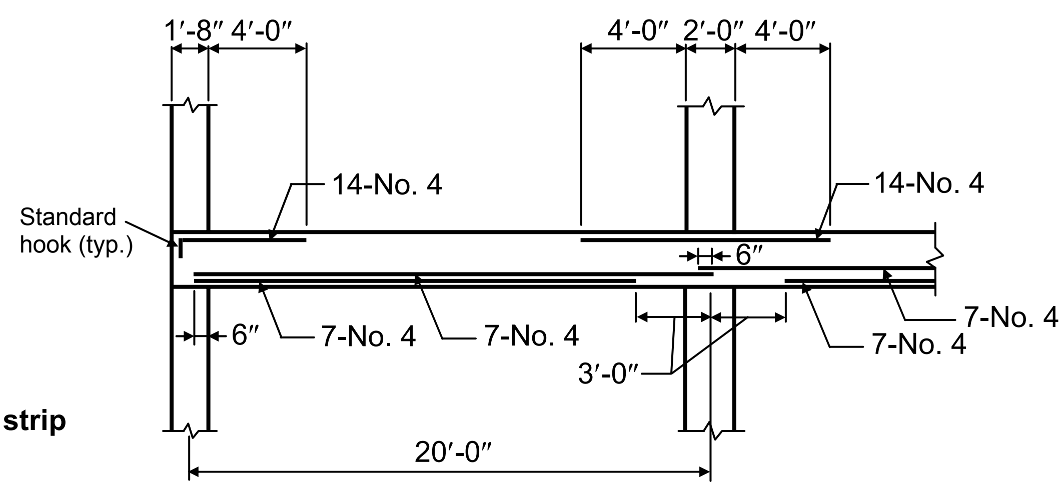
$\phi v_c = 215 \text{ psi} > v_u = 199.4 \text{ psi}$ OK

Reinforcement Details

The figures below show the reinforcement details for the column and middle strips. The bar lengths are determined from Fig. 13.3.8 of ACI 318-99.



Column strip



Middle strip